From Malthus to Modern Growth: The Three Regimes Revisited

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Abstract: This paper sets up a model which replicates the stylized facts of the economic and demographic development of western society the last couple of millennia, as described by Galor and Weil (AER, 1999). Under an initial Malthusian Regime the growth rates of population and income are both low, and the relation between income and population growth is positive. Thereafter follows a Post-Malthusian Regime, where the growth rates of income and population are higher, but the relation between income and population growth is still positive. Finally, the economy transits into a Modern Growth Regime, in which the population growth rate is low, but the growth rate of income is higher than in the Post-Malthusian Regime, and the relation between income and population growth is negative. The model's transition from the first regime to the second follows from a population externality: as the earth becomes more populated transmitting skills form one generation to the next becomes more effective. The transition from the second regime to the third occurs when parental human capital reaches a critical level at which altruistic concerns for children's education becomes operative, triggering a substitution from quantity to quality of children.

1. Introduction

In the history of mankind the level of human population and living standards have remained fairly constant for most of the time. The sudden increase in income that came with the industrial revolution 200 years ago is a very recent phenomenon, and so is the initial rise – and subsequent fall – in population growth that followed it.

This has given birth to a number of thought provoking questions. What caused the industrial revolution in the first place, and why did it take so long time? Was it inevitable? Could it have come at another point in time?

A recent and expanding growth literature takes on this challenge.[1] Within one unified framework, it aims to explain both the stagnant levels of income and population for long periods of time, the recent and sudden rise in income growth (which so far seems to be sustained indefinitely), and the associated changes in population growth.

In particular, the demographic and economic development of the Western World the last couple of millennia has been described by Galor and Weil (1999) as passing through three stages, or regimes. The first is called the *Malthusian Regime*, in which population and standards of living are virtually constant (at low levels with today's standards) or grow very slowly. Also, the relationship between per-capita income and population growth is positive: small increases in income lead to increased population growth. This regime prevails from around year zero to the late 18th century, whereafter the economy transits into what Galor and Weil call a *Post-Malthusian Regime*, with increased growth rates of both population and per-capita income, reflecting that the relationship between population growth and income remains positive, as in the Malthusian Regime. The final stage of the development is the *Modern Growth Regime*, starting towards the end in the 19th century, in which per-capita income spurts, whereas population growth falls, and in some cases becomes negative, reflecting a negative relationship between income and population growth.

Hansen and Prescott (1998) and Jones (1999) model the general transition from stagnation to growth, but do not explicitly capture the three distinct regimes, as described by Galor and Weil (1999). Galor and Weil (2000) provides a model which replicates the stylized facts they outline in their 1999 paper. One of the driving assumptions is that of the interaction between technological progress and the return to education: faster technical change leads to a higher return to education. With a scale effect from population size on technological growth (and many other assumptions; the model is quite rich) they show how the economy transits

[1] An incomplete list includes Hansen and Prescott (1998), Jones (1999), and Galor and Weil (1999, 2000).

through all three regimes.

This paper undertakes on the same task, but in a model which in our view is more simple to understand. The framework is a version of the workhorse model used by Becker, Murphy and Tamura (1990), Ehrlich and Lui (1991), and Tamura (1996). Agents are born with a certain amount of skills, or human capital, and make a decision on how much to educate each child, and how many children to rear. The demographic transition sets in as the non-negativity constraint on education time stops to be binding, and parents start investing time in their offspring, i.e., they substitute quantity for quality. One important novelty is an assumption about the productivity in the human capital sector of the economy, which is assumed to increase with population size, in a manner similar to Galor and Weil (2000).

One interesting and new result is about the inevitability of the industrial revolution. In Galor and Weil (2000) the economy cannot remain in a Malthusian steady state forever, but must transit into a Post-Malthusian phase at some stage. Similarly, in Jones (1999) the industrial revolution is inevitable, although its timing is sensitive to the choice of parameter values. (A change in the fraction of output paid to property right owners is a necessary element for the industrial revolution to take place at all.)

In contrast, our model suggests that the industrial revolution was not at all inevitable, but that initial levels of population and human capital were important. In our setting, the Malthusian Regime is not a steady-state equilibrium [or even a temporary such, as in Galor and Weil (2000)], but rather the opposite: the features of the Malthusian Regime prevails as the economy is located very close to an unstable steady state. For long periods of time, the economy behaves almost as if it was actually located at the very steady state, but as time goes by the unstable features of the system takes overhand. If the economy starts off slightly above the critical levels of population and human capital, it will pass through the three regimes; if it starts off slightly below, it will display fairly constant levels of population and human capital for long periods of time, but eventually population and human capital will fall, and the economy vanish. This is consistent with the findings of Kremer (1993).

The remainder of this paper is organized as follows. Section 2 sets up the model, describing the preferences and the budget constraints. Section 2.3 presents the dynamics: first when education time is constrained to zero (Section 2.3.1), and then when education time operative (Section 2.3.5). Section 3 ends with a concluding discussion, for instance about the role of mortality.

2. The Model

Consider the following overlapping-generations model. Agents live for two periods: childhood, and adulthood. As children, they make no decisions, but impose certain costs for their parents, in terms of consumption goods, and time. The time spent on children builds up their human capital. Agents then enter adulthood, with the human capital built up in the previous period. They allocate time between working in a consumption goods sector, and a human capital sector. They also decide on how many children to rear, given budget constraints for time and goods.

2.1. Budget constraints

Letting all production take place in the household sector, period t output of the consumption good is given by

$$Y_t = Bl_t(L + H_t), \tag{2.1}$$

where Y_t is output, B is a productivity parameter, l_t is time input in the consumption good sector, and $L + H_t$ denotes the time-augmenting human capital. Whereas every agent is born with L units of human capital, regardless of whether or not her parent invested in her human capital, H_t measures the human-capital component inherited from parents.

Goods produced are spent on consumption by the adult and her children:

$$Y_t = C_t + qN_t, \tag{2.2}$$

where C_t is the adult's consumption, q is each child's consumption (exogenous and constant over time), and N_t is the number of children.

The time budget constraint is given by

$$T = l_t + (v + h_t)N_t, \tag{2.3}$$

where T is the total amount of time available for each agent (which is here assumed to be constant over time), and $v + h_t$ is the time spent on each child. The component v is a fixed time cost of rearing a child, which can be interpreted as the time required to rear and nurse children just to keep them alive. The component h_t is the time spent educating each child, which is a choice variable to the parent, and subject to a non-negativity constraint. In other words, parents must spent at least v units of time on each child; if they want to spend less time on children they may reduce either the number of children (N_t), or the education of each child (h_t).

The production function for human capital takes the form

$$H_{t+1} = A_t \left[L + H_t \right] (\rho v + h_t), \tag{2.4}$$

where A_t is a productivity parameter (which is endogenous, but taken as fixed by each atomistic agent; more on that below), and ρv measures the direct inheritance of human capital from one generation to the next, where $\rho \in (0, 1)$. One way to think about this is that the time spent rearing and nursing each child (which the parent is effectively constrained undertake) to some extent adds to the human capital of the child, but not as effectively as the time spend educating the child. For instance, talking and interacting with a child teaches it certain language skills, and a basic acquaintance with the terminology the parent uses. In farming households, children who work with their parents in the fields may accumulate certain skills, without reducing the amount of time parents have available for consumption goods production (l_t). In our setting, this automatic spill-over will be driving growth at early stages of economic development, where $h_t = 0$.

The effectiveness with which one generation transmits its human capital to the next (A_t) is assumed to depend on total population size:

$$A_t = A(P_t), \tag{2.5}$$

where P_t is the population size in period t, which evolves dynamically according to

$$P_{t+1} = P_t N_t. \tag{2.6}$$

We assume the following:

Assumption 1. $A : R_{++} \rightarrow (0, A^*)$ *satisfies*

$$A'(P_t) > 0 \; \forall P_t > 0 \tag{2.7}$$

$$\lim_{P_t \to \infty} A(P_t) = A^* < \infty. \tag{2.8}$$

This "population externality" serves to capture the idea that children learn some of their skills from neighboring households, perhaps in particular from other children. Population density (and thus population size, at a given size of land) should therefore influence the effectiveness with which human capital is transmitted between generations. To keep things simple, we chose to black-box this mechanism, by simply postulating that A_t depends on P_t. One way to model the human capital acquisition process in more detail could be to allow for heterogeneity between households in types of human capital, and let each new generation, in each household, learn some of its skills from neighboring households. The closer are the neighbors, the higher is the aggregate productivity in transmitting human capital between generations.

In the numerical simulations, we shall prevent A_t from falling over time. This serves to rule out falling productivity when the economy reaches the Modern Growth Regime, where population growth is negative. In the simulations, we shall therefore let

$$A_t = \max\{A_{t-1}, A(P_t)\}. \tag{2.9}$$

An intuitive motivation for (2.9) is that the educational institutions which have once been established due to a sufficiently large population will not disappear if the population starts to fall. In the simulations, the Malthusian and Post-Malthusian Regimes will be characterized by growing population, so (2.9) will not matter there. However, if population falls (as it will in the Modern Growth Regime) and human-capital productivity falls with it, that could push the economy back into a new Malthusian Regime, creating cycles, which is not a possibility we want to pursue here.

2.2. Preferences

An adult agent active in period t (referred to as agent t) maximizes a utility function given by

$$U_t = \ln(C_t) + \alpha \ln(N_t) + \alpha\delta \ln(L + H_{t+1}) \tag{2.10}$$

subject to the above budget constraints (2.1) to (2.4). We assume that $\delta \in (0, 1)$ (to guarantee the existence of an interior solution), and that $\alpha > 0$. In (2.10) the second term measures the utility of quantity of children, whereas the third term measures the utility of quality, which is here simply given by the total human capital of the offspring $(L + H_{t+1})$. This could be interpreted as a reduced form of an old-age security motive for rearing children, as in Nishimura and Zhang (1992), and Lagerlöf (1997). Alternatively, one could set up a "dynastic" utility function, where the parent cares about the total welfare of the children, but that would complicate matters due to the non-negativity constraint on h_t, which would make the value function non-differentiable. [This problem has been pointed out by e.g. Tamura (1996, Footnote 11).]

Letting the weight on quality be written as $\alpha\delta$ is simply for notational convenience.

Before maximizing (2.10) it is worthwhile to rewrite the budget constraints (2.1) to (2.3) as

$$C_t = BT\,[L + H_t] - Z_t N_t, \tag{2.11}$$

where $BT\,[L + H_t]$ is "full" income (the amount of the consumption good that a parent would produce if rearing no children), and

$$Z_t = q + (v + h_t)B\,[L + H_t] \tag{2.12}$$

is the total cost per child (consumption goods cost and time cost).

Maximizing (2.10) over the number of children (N_t) subject to (2.11) tells us that

$$N_t = \left(\frac{\alpha}{1+\alpha}\right)\frac{BT\,[L + H_t]}{Z_t}, \tag{2.13}$$

i.e., spending on children is a constant fraction of full income, following from the logarithmic utility. Next, we may substitute (2.13) back into the utility function in (2.10), and use (2.11) and (2.13) to substitute away C_t. Disregarding additive terms, which are taken as constant by agent t (such as H_t), we may write the new utility function as

$$\widetilde{U}_t = \delta \ln\left\{L + A_t\,[L + H_t]\,(\rho v + h_t)\right\} - \ln\left\{q + (v + h_t)B\,[L + H_t]\right\}. \tag{2.14}$$

The first term in (2.14) measures the quality component of children, which the agent can increase by raising education time (h_t); the second term measures the cost of increasing h_t in terms of reduced quantity (N_t), following from higher per-child cost (Z_t).

The first-order condition for h_t tells us that

$$h_t \geq \frac{1}{1-\delta}\left[\frac{q\delta}{B(L + H_t)} + v(\delta - \rho) - \frac{L}{A_t(L + H_t)}\right]. \tag{2.15}$$

The weak inequality in (2.15) follows from the non-negativity constraint on h_t: if the right-hand side of (2.15) is negative $h_t = 0$ (and the inequality is strict); otherwise (2.15) holds with equality.

2.3. Dynamics

To analyze the dynamics of the human capital stock and the population size we distinguish between two situations: one where education time is constrained zero, and one where it is operative. The first case prevails during the Malthusian, and (as the dynamics evolves) Post-Malthusian Regimes; the second during the Modern Growth Regime.

Proposition 1. *$h_t > 0$ if*

$$A_t = A(P_t) > \frac{L}{\frac{q\delta}{B} + v(\delta - \rho)B(L + H_t)}, \tag{2.16}$$

and $h_t = 0$ otherwise.

Proof. Follows directly from (2.15). ■

When h_t is unconstrained, it depends on the parental human capital stock according to

$$\frac{\partial h_t}{\partial H_t} = \frac{1}{1-\delta}\left(\frac{1}{L+H_t}\right)^2 \left[\frac{L}{A_t} - \frac{q\delta}{B}\right]. \tag{2.17}$$

We shall make the following assumption:

Assumption 2.

$$A^* q < BL. \tag{2.18}$$

From Assumption 2 and (2.17) follows that education time is always increasing in the parental human capital stock. This positive relation between the time cost of children and parental human capital is what leads to the negative relation between parental human capital and fertility in the Modern Growth Regime.

2.3.1. The dynamic system when $h_t = 0$

Consider first the case where education time is constrained to zero: $h_t = 0$. We are interested in the dynamic behavior of the population, P, and the human capital stock, H, the latter being a proxy for income, or economic development.

In this section, we shall typically consider equilibria where the population size is growing, and the initial human-capital productivity, A_0, is low enough as to make A_t growing over time. Therefore, it will not matter if we use (2.5) or (2.9): $A_t = A(P_t)$ in both cases. However, the phase diagrams are drawn so that they are formally not consistent with (2.9).

Setting $h_t = 0$, and $A_t = A(P_t)$, in the human capital production function (2.4), we can write

$$H_{t+1} = A(P_t)\left[L + H_t\right]\rho v. \tag{2.19}$$

Similarly for the dynamics of the population in (2.6), using the expression for N_t in (2.12) and (2.13), and imposing $h_t = 0$, we can write

$$P_{t+1} = P_t N_t = P_t \left(\frac{\alpha}{1+\alpha}\right) \frac{BT\left[L + H_t\right]}{q + vB\left[L + H_t\right]}. \tag{2.20}$$

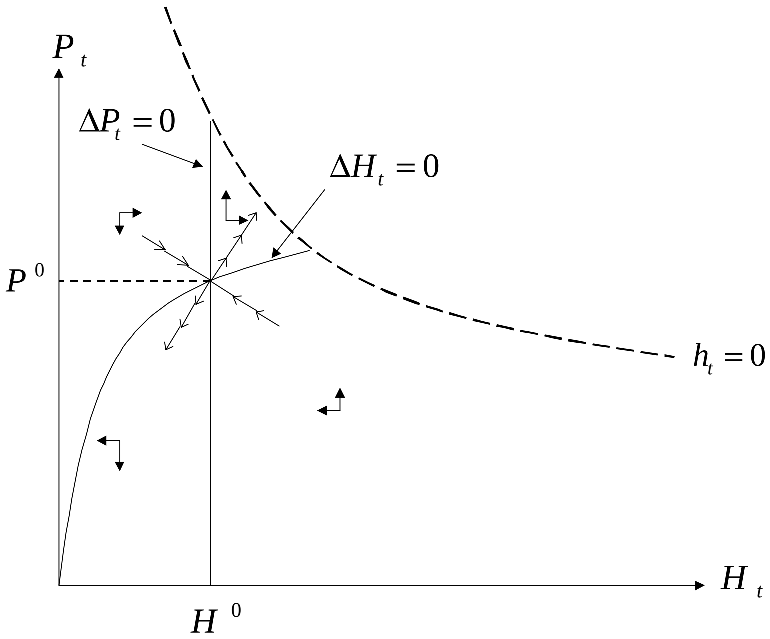

Figure 2.1: The phase diagram when $h_t = 0$ (below the dashed line).

The dynamics of the economy, when $h_t = 0$, is described by the two-dimensional system of difference equations (2.19) and (2.20). This is analyzed in the phase diagram in Figure 2.1. We begin by deriving the loci along which human capital and population are constant.

2.3.2. The $\Delta H_t = 0$ locus

Imposing $H_{t+1} = H_t$ in (2.19) we can write the $\Delta H_t = 0$ locus as

$$P_t = A^{-1}\left(\frac{H_t}{\rho v\left[L + H_t\right]}\right). \tag{2.21}$$

When P_t exceeds the right-hand side of (2.21), H is growing over time, and when P_t falls below the right-hand side H is falling, as illustrated in Figure 2.1 below.

Clearly, the $\Delta H_t = 0$ locus slopes upwards since

$$\frac{\partial P_t}{\partial H_t}\mid_{\Delta H_t=0} = \frac{1}{\rho v A'(P_t)}\left(\frac{1}{L + H_t}\right)^2 > 0. \tag{2.22}$$

2.3.3. The $\Delta P_t = 0$ locus

(2.20) tells us that the population is constant where the human-capital stock equals

$$H^0 \equiv \frac{q/B}{\left(\frac{\alpha T}{1+\alpha}\right) - v} - L. \tag{2.23}$$

We impose the following parametric restriction to make sure that H^0 is positive:

Assumption 3.

$$\frac{q/B}{\left(\frac{\alpha T}{1+\alpha}\right) - v} > L \tag{2.24}$$

Figure 2.1 shows the $\Delta P_t = 0$ locus, which is a vertical line. As shown, fertility is above unity (implying that the population is growing) whenever $H_t > H^0$, and below unity when $H_t < H^0$.

2.3.4. The transition from a Malthusian to a Post-Malthusian Regime

The intersection of the $\Delta P_t = 0$ locus and the $\Delta H_t = 0$ locus occurs at the point (H^0, P^0), which constitutes a saddle-path stable steady-state equilibrium. An economy which starts off exactly there will remain there for eternity.

An economy which starts off at a slightly higher initial level of population and human capital will ostensibly be in a steady-state equilibrium at early stages of development. Since it is situated so close to the steady-state, the changes in human capital and population size are virtually zero for long periods of time, which is exactly what characterizes the Malthusian Regime. This may continue for many generations, but eventually as the economy slowly moves upwards and to the right (i.e., as the population size, and the levels human capital slowly increase) the speed of change accelerates. Population and human capital start growing faster, and the economy moves from a Malthusian to a Post-Malthusian Regime.

What drives this transmission is the explosive characteristics of the dynamic system in (2.19) and (2.20). As population increases, so does the productivity of human-capital production, which raises the income level of the next generation. This in turn feeds back into increased demand for children, raising the population size even further, and thus human-capital productivity.

The fact that the economy remains at an almost constant level of population and human capital for a long period of time is an important part of the story. As

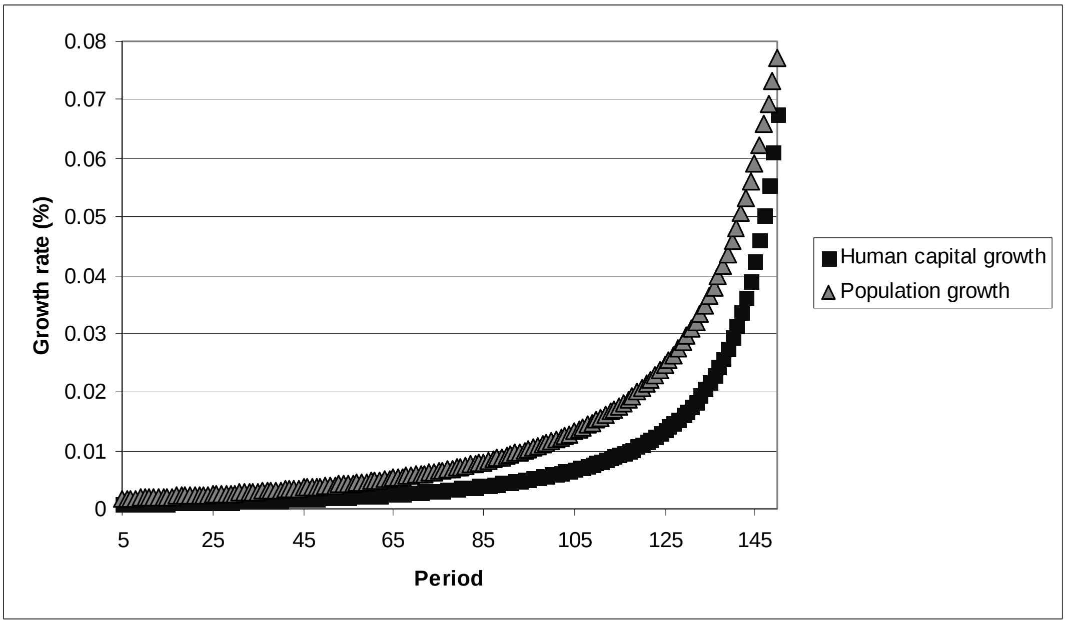

Figure 2.2: The transition from a Malthusian regime to a Post-Malthusian.

the numerical illustration in Figure 2.2 shows[2], both population growth $(N_t - 1)$, and human capital growth $(H_{t+1}/H_t - 1)$, remain around zero for a long period of time, only to simultaneously explode after a large number of periods. Simply the fact that the economy starts off close to an unstable steady-state equilibrium gives this prediction: nothing happens for long periods of time, but then, at some stage, the unstable features of the system takes overhand, and pushes the economy onto an explosive trajectory.

Similarly, as can be seen in Figure 2.1, economies starting off slightly *below* the critical levels of population and human capital (H^0, P^0) will initially show similar behavior as those starting off slightly above. Growth rates will be around zero for a long period of time. Eventually, however, they contract into a zero human-capital steady state, with vanishing population. This is consistent with the finding of Kremer (1993), that larger initial population size leads to faster technological change and population growth.

[2]The parameter values are identical to those in Section 2.3.10 below.

2.3.5. The dynamic system when $h_t > 0$

Consider the explosive path in the phase diagram in Figure 2.1 again. As can be seen from Proposition 1, the accelerating growth rates of population and human capital must eventually make education-time operative, namely when (2.16) starts to hold. In terms of Figure 2.1, this happens when the trajectory that the economy is following intersects the $h_t = 0$ frontier.

When education time is operative, the dynamics of the human capital stock is given by the human capital production function (2.4), substituting for the optimal h_t, as given by (2.15) holding with equality. After some algebra, this gives

$$H_{t+1} = \frac{A(P_t)}{1-\delta}\left[\frac{\delta q}{B} - \frac{L}{A(P_t)} + v\delta(1-\rho)\left[L + H_t\right]\right]. \tag{2.25}$$

Similarly, after substituting the expression for optimal h_t in (2.15) into the expressions characterizing optimal fertility in (2.12) and (2.13), some amount of tedious algebra tells us that

$$N_t = \left(\frac{\alpha}{1+\alpha}\right)\frac{(1-\delta)BT\left[L+H_t\right]}{q - \frac{BL}{A(P_t)} + v(1-\rho)B\left[L+H_t\right]}, \tag{2.26}$$

where we note that fertility is now falling in the parental human capital stock:

$$\frac{\partial N_t}{\partial H_t} = \left(\frac{\alpha(1-\delta)BT}{1+\alpha}\right)\frac{q - \frac{BL}{A(P_t)}}{\left\{q - \frac{BL}{A(P_t)} + v(1-\rho)B\left[L+H_t\right]\right\}^2} < 0. \tag{2.27}$$

The inequality follows from Assumption 1, which guarantees that higher parental human capital leads to higher demand for quality (education time), and therefore lower demand for quantity.

From (2.26) and (2.20) the population dynamics can be written

$$P_{t+1} = P_t\left(\frac{\alpha}{1+\alpha}\right)\frac{(1-\delta)BT\left[L+H_t\right]}{q - \frac{BL}{A(P_t)} + v(1-\rho)B\left[L+H_t\right]}. \tag{2.28}$$

The dynamics of the economy when $h_t > 0$ is characterized by the two-dimensional system of difference equations in (2.25) and (2.28). To keep the analysis comparable to that of the previous section, we begin by allowing A_t to be decreasing, i.e., we let A_t be given by (2.5) rather than (2.9). Once we have seen where this leads us, we shall assume that A_t cannot decrease, by letting its dynamics be given by (2.9).

2.3.6. The $\Delta H_t = 0$ locus

Setting $H_{t+1} = H_t$ in (2.25), and solving for P_t, we can write the $\Delta H_t = 0$ locus as

$$P_t = A^{-1}\left\{\frac{L + (1-\delta)H_t}{\frac{\delta q}{B} + v\delta(1-\rho)\left[L + H_t\right]}\right\}. \tag{2.29}$$

The slope of this locus depends on the sign of $(1-\delta)(q/B) - v(1-\rho)L$, and will not be essential for the rest of the analysis. (We have drawn it with positive slope in the diagram in Figure 2.3 below.) Note that the locus converges to $A^{-1}\left(\frac{(1-\delta)}{v\delta(1-\rho)}\right)$ as H_t goes to infinity.

Positions above this locus imply growing levels of human capital, and positions below imply falling levels.

2.3.7. The $\Delta P_t = 0$ locus

Setting $P_{t+1} = P_t$ (i.e., $N_t = 1$) in (2.28), and solving for P_t, gives the $\Delta P_t = 0$ locus:

$$P_t = A^{-1}\left\{\frac{BL}{q + \omega B\left[L + H_t\right]}\right\}, \tag{2.30}$$

where

$$\omega = v(1-\rho) - \left(\frac{\alpha}{1+\alpha}\right)(1-\delta)T. \tag{2.31}$$

Positions above this locus imply falling population, and positions below imply growing population.

We also make the following assumption:

Assumption 4.

$$\omega > 0. \tag{2.32}$$

Assumption 4 makes the steady-state fertility rate in the Modern Growth Regime less than unity, implying a falling population, which turns out to give the predictions we want in the numerical simulations. It also makes the argument in $A^{-1}(\cdot)$ in (2.30) approach zero as H_t goes to infinity. For most specifications of $A(\cdot)$, this is needed to guarantee that fertility actually falls in the Modern Growth Regime.

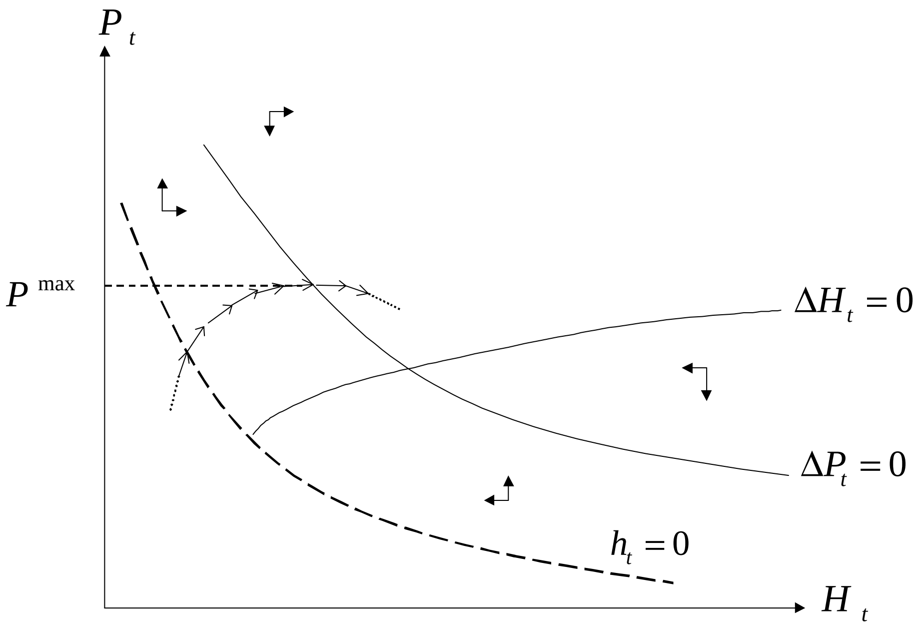

Figure 2.3: The phase diagram when h_t >0 (above the dashed line).

2.3.8. The transition to Modern Growth

The phase diagram in Figure 2.3 demonstrates the dynamics of the economy from the point at which education time becomes operative, i.e., when the explosive trajectory in Figure 2.1 intersects the $h_t = 0$ frontier. Note that the $\Delta H_t = 0$ locus in the case where $h_t = 0$ must coincide with the $\Delta H_t = 0$ locus in the case where $h_t > 0$ exactly where they intersect the $h_t = 0$ frontier. Since the explosive trajectory from Figure 2.1 must lie above the $\Delta H_t = 0$ locus in Figure 2.1, it must break through the $h_t = 0$ frontier in a section above the $\Delta H_t = 0$ locus in Figure 2.3, just as drawn.

In other words, once education time has become operative, both population and human capital continues to grow. However, population grows at a slower and slower pace, until it eventually intersects the $\Delta P_t = 0$ locus, and starts to fall.

Human capital, on the other hand, grows at an increasing rate, until eventually the lower population size spills over to lower A_t. This stops the growth of human capital, which starts to fall as the economy intersects the $\Delta H_t = 0$ locus, after which both human capital and population are falling.

As human capital falls, fertility goes up [see (2.27)], and eventually population starts increasing, pushing A_t upwards again. This eventually makes human capital start growing, as the trajectory intersects the $\Delta H_t = 0$ locus once again, which completes one round of a cyclical pattern, the beginning of which is sketched in Figure 2.3.

2.3.9. Irreversible A_t

In short, this cyclical behavior stems from the fact that the population decline in the Modern Growth Regime pushes the economy back in time, forcing it to repeat demographic transition, and growth spurt, over and over again. The key element is the human capital productivity parameter, A_t, which starts falling with population. However, there are good reasons to believe that improvements in education productivity are not that easily reversed. For instance, certain techniques and skills could make population size (and population density) itself less crucial for human capital transmission between generations. Transport and communication improvements, as well as changes in the geographical location of the population, could make the very dependency of A_t upon P_t obsolete. For instance, the alphabet, book printing, telephones, and the internet all tends to make the world smaller. Also, as more people chose to live in cities, educational institutions (universities, colleges, etc.) should more easily find a necessary critical mass of knowledge, and students.

To capture this irreversibility in human capital productivity, we can let the dynamics of A_t be given by (2.9), so that A_t cannot fall below its level in the previous period. This gives us three-dimensional system of difference equations: one each for H_t, and P_t, as before, and yet third one for A_t. The first two are simply (2.25) and (2.28), with $A(P_t)$ is replaced by A_t. The third one is (2.9), forwarded one period, and substituting the expression for P_{t+1} into the second argument in the maximization expression:

$$A_{t+1} = \max\left(A_t, A\left\{\underbrace{P_t\left(\frac{\alpha}{1+\alpha}\right)\frac{(1-\delta)BT\left[L+H_t\right]}{\left[q-\frac{BL}{A_t}\right]+v(1-\rho)B\left[L+H_t\right]}}_{P_{t+1}}\right\}\right). \tag{2.33}$$

As long as the population is growing (and the initial level of A is low enough) A_{t+1} will be given by the second argument in the maximization expression, and the dynamics are the same as those described above, with reversible A. When the population starts to fall however, A stays constant over time at

$$A^{\max} = A(P^{\max}), \tag{2.34}$$

where $P^{\max}$ is the size of P_t just before it starts declining (see Figure 2.3). Replacing $A(P_t)$ in (2.25) with this constant $A^{\max}$ gives a one-dimensional difference equation for human capital:

$$H_{t+1} = \frac{1}{1-\delta}\left(\frac{\delta q A^{\max}}{B} - L\left[1 - v\delta(1-\rho)\right]\right) + \left(\frac{v\delta(1-\rho)A^{\max}}{1-\delta}\right) H_t, \tag{2.35}$$

where we note that the intercept term is positive from Assumptions 1 and 2. Human capital will be growing over time if $v\delta(1-\rho)A^{\max} > 1-\delta$ (as confirmed by drawing a standard 45-degree diagram). Fertility will keep falling as human capital grows, and eventually converge to

$$\begin{aligned} \lim_{H_t\to\infty} N_t &= \lim_{H_t\to\infty}\left(\frac{\alpha}{1+\alpha}\right)\frac{(1-\delta)BT\left[L+H_t\right]}{q - \frac{BL}{A^{\max}} + v(1-\rho)B\left[L+H_t\right]} \\ &= \frac{\alpha(1-\delta)T}{v\left(1+\alpha\right)\left(1-\rho\right)} < 1 \end{aligned} \tag{2.36}$$

where the inequality comes from Assumption 4.

To sum up, once education time has become operative, population starts growing slower, eventually reaches its maximum, and then starts falling. From this point in time, A remains constant. Human capital continues to grow at a steady rate while the population falls, and eventually vanishes.

2.3.10. Simulations

Figure 4 shows a numerical example of how the dynamics of human capital and population looks as the economy transits from a Malthusian Regime, via a Post-Malthusian Regime, to a Modern Regime. The first part is identical to Figure 2.2, and the continuation shows where that explosive path eventually leads.

We choose to specify the function for the productivity of the human capital sector as

$$A(P_t) = \frac{A^* P_t}{1+P_t}, \tag{2.37}$$

which clearly satisfies Assumption 1. The remaining parameter values are chosen as in Table 2.1.

The particular numerical example is not chosen to replicate any quantitative features of data, but rather to give an overview of the results of the model.

The main result is captured by Figure 2.4, which shows the population growth rate ($N_t - 1$) and human capital growth rate ($H_{t+1}/H_t - 1$), from period 5 to 200.

Parameter	ρ	δ	α	T	v	q	B	L	A^*
Value	.7	.75	1	1	.45	17.5	1	300	15

Table 2.1: Numerical values

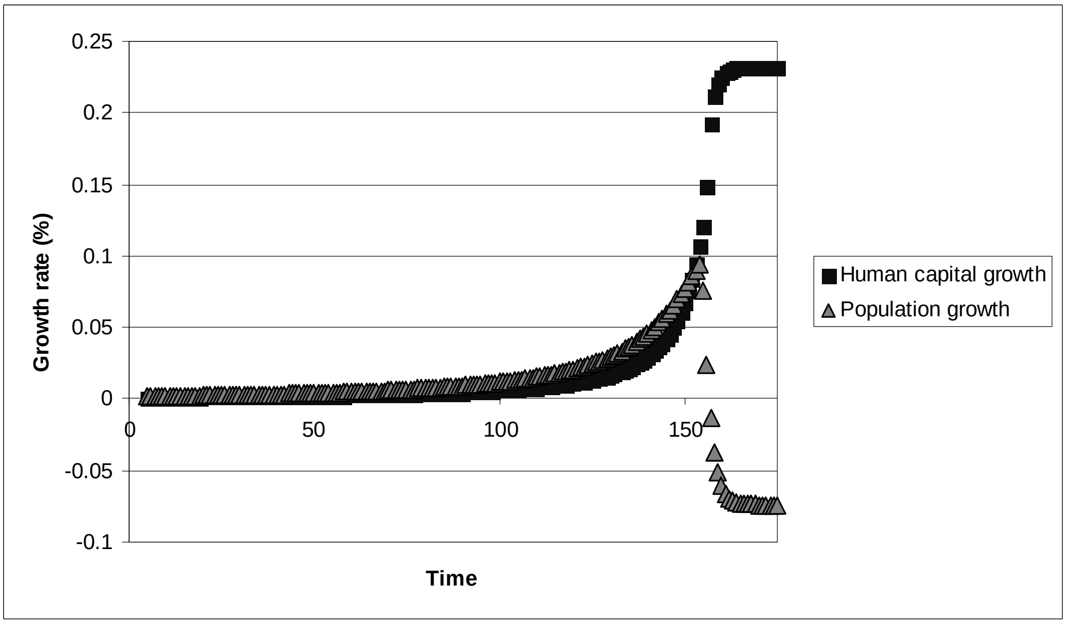

Figure 2.4: The whole transition from a Malthusian Regime, via a Post-Malthusian, to a Modern Growth regime.

We let the economy start off with a population size, and a human capital stock, 10% above their respective critical levels (P^0 and H^0). The first couple of periods the economy moves to the exploding trajectory, as illustrated in Figure 2.1, which is why we start counting from period 5.

As seen, both population and human capital growth rates lie fairly constant around zero for the first 100 periods. This is the Malthusian regime. Then they both start increasing, up until approximately period 150. This is the Post-Malthusian Regime. Suddenly, in period 155, population growth drops (from almost 10% to negative, in this numerical example), whereas human capital growth shoots up (from around 10%, to around 23%, in this example). After about 5 periods, the economy has stabilized at high positive human capital growth, and negative population growth. This is the Modern Growth Regime.

3. Conclusions

The aim of this paper is to replicate the stylized facts of the economic and demographic development of western society the last couple of millennia, as described by Galor and Weil (1999). Under an initial Malthusian Regime the growth rates of population and income are both low, and the relation between income and population growth is positive. Thereafter follows a Post-Malthusian Regime, where the growth rates of income and population are higher, but the relation between income and population growth is still positive. Finally, the economy transits into a Modern Growth Regime, in which the population growth rate is low, but the growth rate of income is higher than in the Post-Malthusian Regime, and the relation between income and population growth is negative.

The model we use is a version of that used by Becker et al. (1990), Ehrlich and Lui (1991), and Tamura (1996). Agents are born with a certain amount of human capital, and decide on how much to educate each child, and how many children to rear. Education time is constrained to zero in the Malthusian and the Post-Malthusian regimes. Therefore, due to a consumption goods cost of rearing children, increased parental human capital leads to higher demand for children, which expands the population size.

Moreover, the productivity in the human capital sector of the economy is assumed to increase with population size, in a manner similar to Galor and Weil (2000). This makes the dynamics of the economy explosive. However, an economy starting off very close to an unstable steady state will remain at a virtually constant population size through a long period of time. This is the Malthusian Regime. Eventually the unstable features of the system sets in and the economy transits to a Post-Malthusian Regime.

The transition into the Modern Growth Regime sets in as the non-negativity constraint on education time stops to be binding, and parents start investing time in their offspring, i.e., they substitute quantity for quality. The growth rates shoots up, and fertility falls to negative numbers.

Different from most earlier work, our model suggests that the industrial revolution was not at all inevitable. Because if the initial levels of population and human capital are slightly too low, the economy is doomed to vanish. However, if it starts off close enough to the critical values, for a long time it will be hardly distinguishable from an economy which starts off slightly above the critical values, and is set to go through an industrial revolution. This is in line with the observation of Kremer (1993), that initial population size has a positive effect on subsequent population growth. It also explains why so many civilizations have vanished from earth.

The results of this paper would be strengthened if we allowed for changes in mortality. Letting the average, or aggregate, human capital stock influence the longevity parameter T, the explosive behavior of the economy would be reinforced. As T increases, parents have more effective time, which increases demand for children, and raises population size. This spills over to higher human capital productivity, and increasing levels of human capital across generations, feeding back to the longevity parameter T. Assuming that there is an upper bound on T, this effect would vanish at some stage of economic development. This would first make mortality fall (as longevity increases), together with increasing fertility, whereafter mortality flattens out, and fertility falls as education time becomes operative. We have chosen to leave this mechanism out of the model simply to make it easier to understand.

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